

Calculation for guided waves in pipes and rails

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Abstract. This paper describes a calculation technique with a semi-analytical finite element method for guided waves and its application to simulation and modal analysis of wave propagation in a pipe and a bar with an arbitrary cross-section as rail. Dispersion curves and wave structures for any kinds of bar like structures can be calculated by the SAFEM. This study examines dispersion curves for a square bar and a rail. Also, visualization results of guided wave propagation are shown for a straight pipe, a pipe with an elbow, and a pipe with a spherical defect.

Introduction

Ultrasonic guided waves are a type of wave propagation in which the wave is guided in plates, rods, pipe or elongated structures such as rails and I beams. Recently, long-range inspection of pipes by the use of guided waves has attracted considerable attention because this technique largely reduces inspection time and costs compared to the ordinary point-by-point testing in large pipeworks [1]-[3]. Especially in Japan, since a large number of pipeworks constructed 30 years ago in highly economic era are aging and require maintenance work or replacements, the inspection technique for large structures has become a very urgent subject.

Commercial equipments with guided waves are installed at one location of a pipe and reflection echoes analyzed indicating the presence of corrosion or other defects. In guided wave inspection of a pipe with elbows and defects or a bar with an arbitrary cross-section, however, guided waves propagate with very complicated wave structures due to a mixture of multi-modes and mode conversions, which prevents guided waves from being widely used.

Modal analysis and simulation of guided wave propagation with computations are very useful for solving lots of problems due to the complexity of guided wave. The authors have carried out guided wave calculations with a semi-analytical finite element method (SAFEM)[4]-[6]. Since the SAFEM does not require divisions in the longitudinal direction, long-range calculations can be done with no problems of calculation time and memory. Modal analysis is also possible, because guided wave propagation is calculated as a summation of resonance modes in the SAFEM.

This paper describes several applications of the SAFEM applied to guided wave simulation and analysis. After brief description about the SAFEM, calculation results on guided waves in a pipe and a bar with an arbitrary cross-section are presented. Dispersion curves and wave structures for a square bar and a rail are discussed, and simulation results of wave propagation in a pipe with an elbow and a defect are visualized.

The semi-analytical finite element method for guided wave propagation

Guided waves have a potential of long-range inspection in the meter or hundred-meter order, which is significantly larger than an ultrasonic wavelength. Using ordinary finite element or boundary element methods, therefore, extremely large calculation times and memory are required for calculations of guided wave propagation in a pipe and a rail. Since a semi-analytical finite element method (SAFEM)

does not require discretization in the propagation direction of the guided waves, it is very useful for long-range guided wave calculations.

The cross-sections of a pipe and a bar are discretized into small sections as shown in Fig.1. Instead of dividing the region in the longitudinal direction as in ordinary FEM, the orthogonal function $\exp(i\xi z)$ is used for expressing the distribution of the displacement field in the longitudinal direction. Similarly to ordinary FEM, the virtual work principle or minimization of potential energy in the entire volume of an object rewrites the governing equations into an integration form. Discretizing the integration form then gives an eigensystem with respect to the wave number ξ for a certain frequency. Eigenvalues and eigenvectors obtained from the eigensystem correspond to wave numbers and wave structures for resonance guided wave modes in the bar-like object. For given boundary conditions, displacement fields can be obtained as a summation of guided wave modes. Wave propagation can be simulated by collecting these displacement data for all frequency steps in the frequency bandwidth.

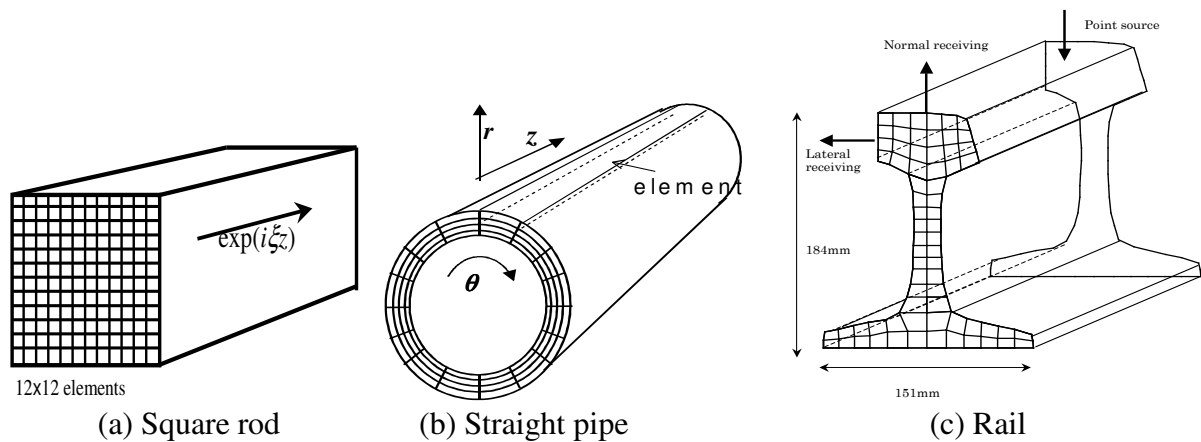


Fig. 1. Sub-divisions in the semi-analytical finite element method

Results and discussions

Dispersion curves and wave structures. Dispersion curves are obtained for a pipe and a bar with an arbitrary cross-section by solving the eigensystem at all frequency steps, as well as the wave structures for corresponding modes. Fig.2 shows phase velocity and group velocity dispersion curves for a square rod with the Poisson ratio of 0.3 as shown in Fig.1(a). Also Fig.3 shows the wave structures at the point indicating in the dispersion curves of Fig.2. Fig.3(a) and (d) are the bird's eye-view and (b) and (c) are the front view of the cross-section so as to see the wave structures easily. Wave structures are totally different at the different points in the dispersion curves, in which (a) and (d) are longitudinal vibrations, and (b) is a flexural mode and (c) is a torsional mode. The velocity of the longitudinal mode (a) is a little bit smaller than the longitudinal wave speed c_L and close to the plate

wave speed $c_{plate} = 2c_T \sqrt{1 - \left(\frac{c_T}{c_L}\right)^2}$. The group velocity of (d) is a little smaller than that of (a). However,

considering that most of dispersion curves are below $c_g/c_l=1$, the mode at (d) is relatively fast. The dispersion curve of mode (b) is very similar to that of the A0 mode of the Lamb wave, and the wave structure shows the flexural vibration like an A0 mode. The torsional mode (c) has slightly smaller velocity than the transverse wave velocity c_T .

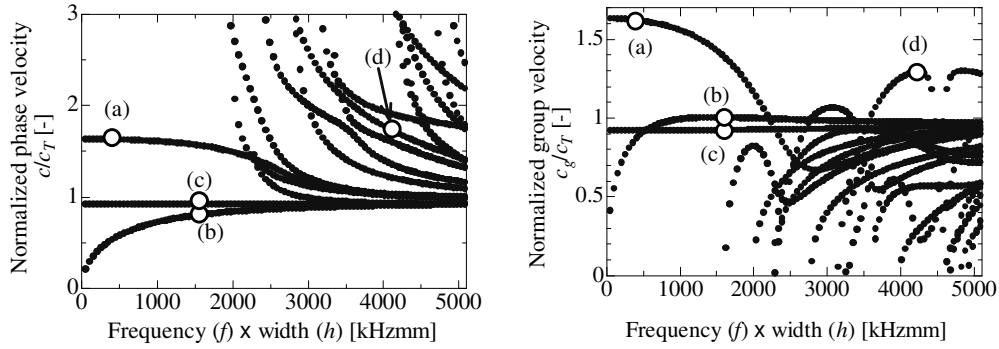


Fig. 2. Dispersion curves for a square rod of h (mm) \times h (mm).

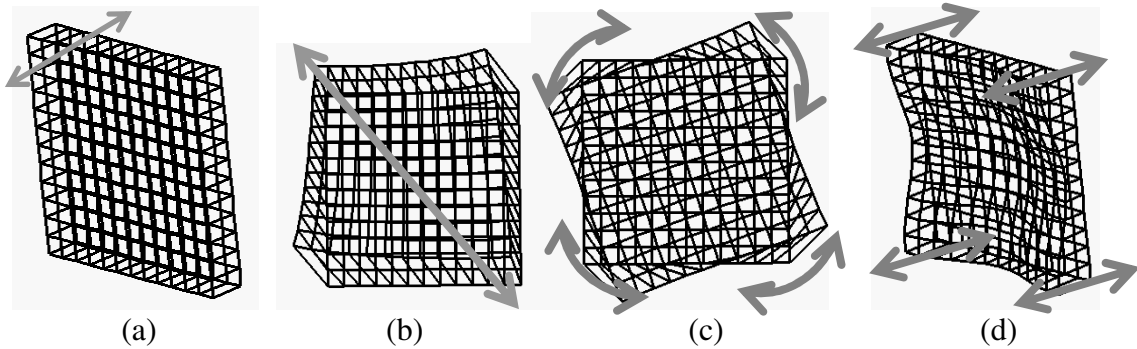
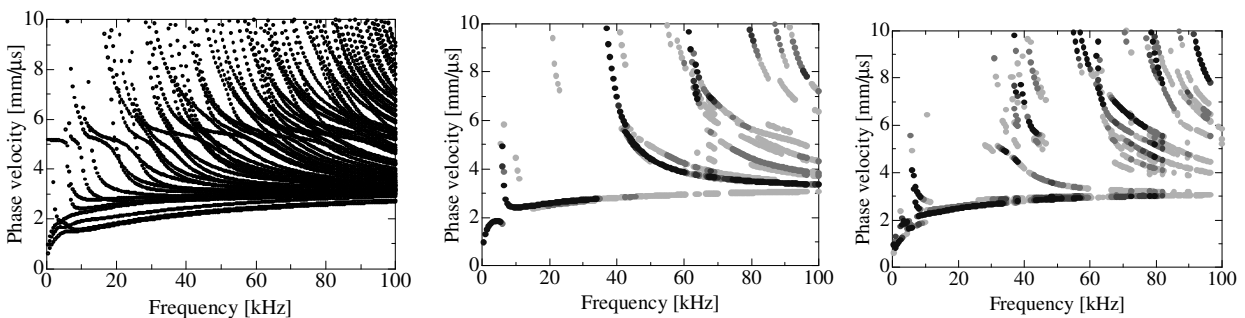


Fig. 3. Wave structures for the different regions shown in the dispersion curves of Fig.2.

Similarly, dispersion curves for a rail can be obtained by the SAFEM as shown in Fig.4(a). Fig.3(c) shows the sub-divisions for the SAFE calculation. Since the dispersion curves in Fig.4(a) includes all possible resonance modes, many unnecessary modes are shown under certain conditions. Fig.4(b) and (c) are phase velocity dispersion curves where dominant modes are highlighted under measurement conditions for normal displacements on the upper (b) and lateral (c) surfaces of the rail head are detected for normal incidence on the upper surface. These two dispersion curves show the characteristics of normal and lateral vibrations of the railhead. The dispersion curves considering measurement conditions agreed well with the experimental results obtained by two dimensional FFT technique [6].



(a) all possible resonant modes (b) detected on the upper surface (c) detected on the lateral surface

Fig. 4. Dispersion curves for a rail shown in Fig.1(c).

(b) and (c) show only dominant modes under different measurement conditions.

Simulation of wave propagation in a pipe. The semi-analytical finite element technique gives numerical data sets of displacements and stresses at any points in a pipe at every time step. A visualization software, Micro AVS (KGT. Inc.), can convert the data set into a video clip as shown in Fig.5 and 6. Formulation and detailed calculation techniques are written by the authors [5, 6].

Figure 5 shows the propagation status of the axisymmetric mode in a straight pipe. Axisymmetric loading is applied at the left edge of the pipe with 3-inch diameters. The grid shift and shading shows the absolute value and real value of the complex amplitude, respectively. As a result, $L(0,1)$ and $L(0,2)$ modes are excited and propagate toward the right. Since they have different group velocities, these two modes separate as they travel.

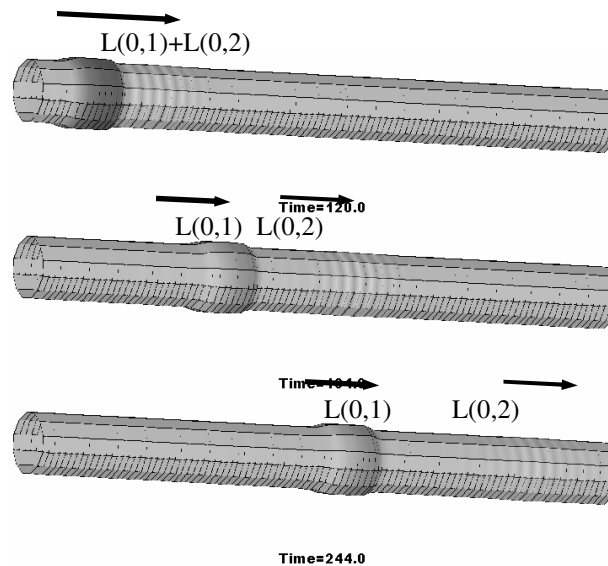


Fig. 5. Axisymmetric guided wave in a straight pipe, showing two possible axisymmetric modes with different velocities.

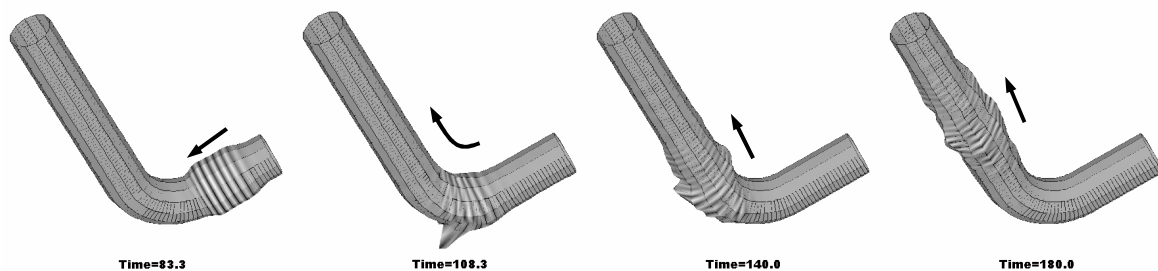


Fig. 6. Axisymmetric wave input in a pipe with elbow, showing the wave break-up and mode conversion at an elbow and subsequent non-axisymmetric wave propagation.

Wave propagation in a pipe with an elbow region can also be calculated by combining the two semi infinite straight regions and one curved region. A quasi cylindrical coordinate system with a curved z axis is used in the curved region, and these regions are combined with the continuity of displacements and stresses. Fig.6 shows wave propagation when the axisymmetric modes are excited in a pipe with an elbow. Axisymmetric modes largely distorted at the elbow, and with large amplitudes can be seen at the outskirts of the elbow. Subsequent waves become very complex and have relatively small amplitude. This causes small and complex reflection echo from defects, and makes pipe inspection difficult. Therefore, new guided wave focusing and tuning technique will be necessary. [5]

The combination technique of the SAFEM and the other modeling methods such as an FEM and a BEM is also possible. This technique establishes the model of a pipe with an arbitrary shape defect. Fig.7 shows scattering from a spherical defect on a pipe when an axisymmetric torsional mode $T(0,1)$ of 50 kHz center frequency as input. Fig.8 shows the dimensions of the pipe used in this calculation. The shadings on the surfaces and the grid lines mean displacements in the r and θ directions, respectively. The spherical defect can be seen on the top of the pipe in Fig.7 (a). The input $T(0,1)$ mode propagates into the crack region as shown in Fig.7(b), and after the displacement in the r direction is excited in Fig.7(b), three dark regions are seen in the circumferential direction, showing

the T(2,1) mode. Since the T(2,1) mode is very dispersive and slow at the frequency region used here, the T(2,1) mode can separate from the other modes as T(0,1) and T(1,1).

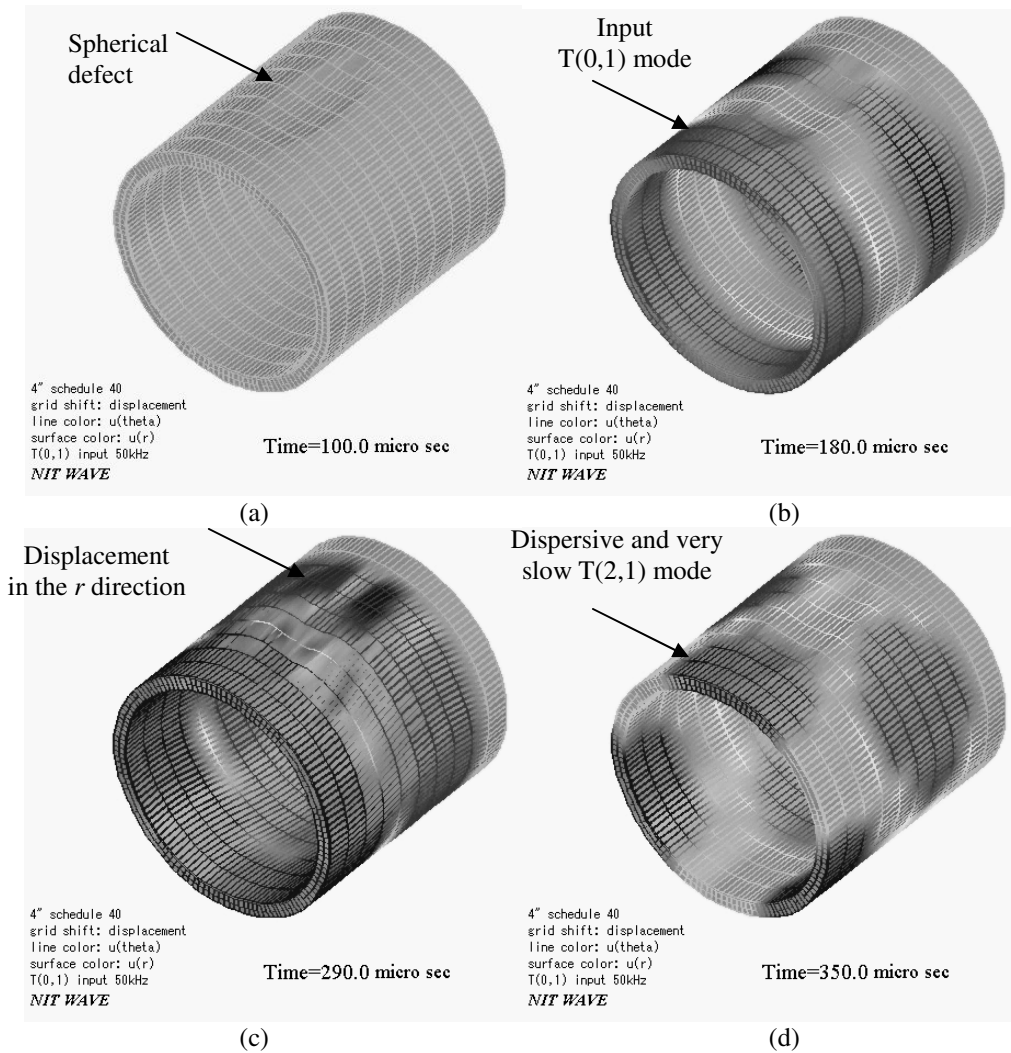


Fig. 7. Scattering wave from a spherical defect on a pipe.

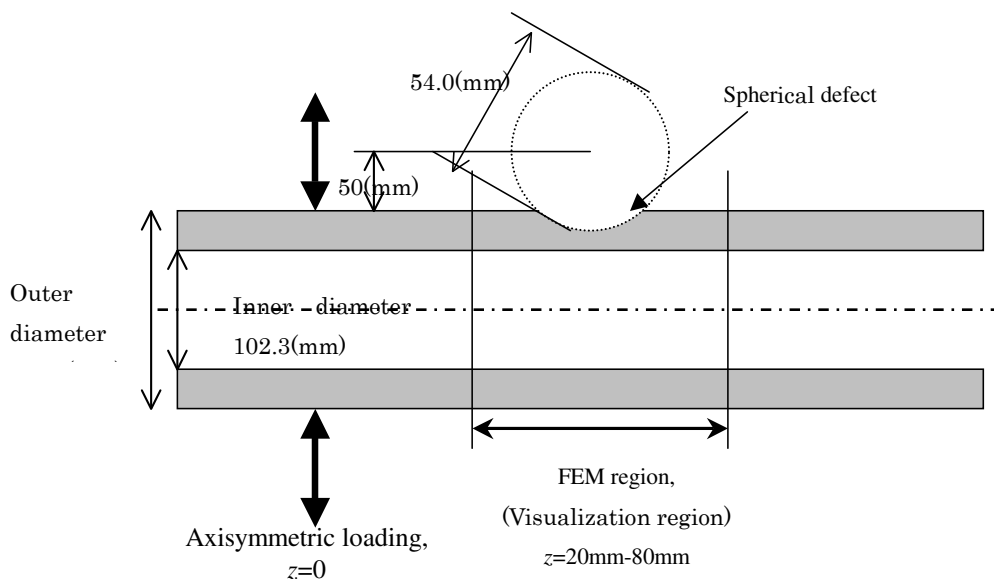


Fig. 8. Dimension of a pipe with a defect

Conclusions

A calculation technique by the semi-analytical finite element method was briefly described, and guided wave calculations were presented on dispersion curves and wave structures, and simulations of wave propagation in a straight pipe, a pipe with an elbow and a pipe with an spherical defect.

Guided wave calculations lead to new knowledge with an explanation to observations that arise in experiments. The authors have already developed the SAFEM codes for many different applications of guided wave analysis as shown in this paper. In the next step, we need to provide software so that researchers and technicians can use guided waves in wider applications.

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